Neutrino Quantum Kinetic Equations:

the Collision Term

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Overview

- **Motivation**: Impact of neutrino interactions in supernovae, BH accretion-discs, neutron-star mergers
- **Background**: Review QKEs
- **Collision terms**: Results of work in progress

Motivation

Good description of neutrino interactions

- Neutrino interactions impact abundance of heavy elements in neutrino driven winds in supernovae, BH accretion-discs, neutron-star mergers
- Needed for complete description of neutrino transport in early universe, core collapse supernovae and compact mergers

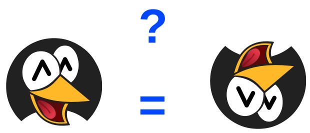
SN 1994D, NGC 4526 galaxy

- Quantum Kinetic Equations (QKE): evolution of ensemble of neutrinos in hot dense media
- Closed Time Path formalism for non-equilibrium QFT

Why QKE?

- Account for **kinetic**, **flavor** and **spin** degrees of freedom: study interaction of all flavors of neutrinos with electrons, protons, neutrons
- More detailed than mean field approach
- Anisotropic regions: spin-flip yields
  - Neutrino-antineutrino transformation for Majorana neutrinos
  - Active-sterile transformation for Dirac neutrino
  - QKEs depend on absolute mass scale

→ *Possible path to distinguishing between Majorana vs Dirac neutrinos*



Further details: Cirigliano, Fuller, Vlasenko, *Phys.Lett.* **B747** (2015) 27; Serreau, Volpe, *PRD* **90** (2014) 125040

Background

Effective interactions

Assume neutrino energy below electroweak scale (<< 100 GeV), effective Lagrangians after integrating out W,Z:

$$\mathcal{L}_{\nu\nu} = -\frac{G_F}{\sqrt{2}}\; \bar{\nu}\gamma_\mu P_L \nu \;\; \bar{\nu}\gamma^\mu P_L \nu ,$$

$$\mathcal{L}_{\nu e} = -2\sqrt{2} G_F \left(\bar{\nu}\gamma_\mu P_L \underline{Y_{eL}} \nu \;\; \bar{e}\gamma^\mu P_L e \; + \; \bar{\nu}\gamma_\mu P_L \underline{Y_{eR}} \nu \;\; \bar{e}\gamma^\mu P_R e \right)$$

$$\mathcal{L}_{\nu N} = -\sqrt{2} G_F \sum_{N=p,n} \bar{\nu}\gamma_\mu P_L \nu \;\; \bar{N}\gamma^\mu \left(\underline{C_V^{(N)} - C_A^{(N)}\gamma_5} \right) N ,$$

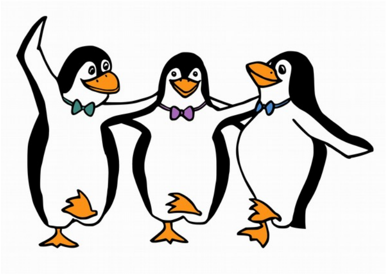
$$\mathcal{L}_{CC} = -\sqrt{2} G_F \; \bar{e}\gamma_\mu P_L \nu_e \; \bar{p}\,\gamma^\mu \left(1 - \underline{g_A}\gamma_5\right) n \; + \; \text{h.c.}$$

$$P_{L,R} = (I \mp \gamma_5)/2 , \qquad \nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} .$$

Effective interactions

… with electron couplings

$$Y_{eL} = \mathrm{diag}\left(\tfrac{1}{2}+\sin^2\theta_W, -\tfrac{1}{2}+\sin^2\theta_W, -\tfrac{1}{2}+\sin^2\theta_W\right) ,$$
$$Y_{eR} = \sin^2\theta_W \times I \ .$$



Nucleon couplings:

$$C_V^{(p)} = \frac{1}{2} - 2\sin^2\theta_W \ , \qquad C_V^{(n)} = -\frac{1}{2} \ ,$$
$$C_A^{(p)} = \frac{g_A}{2} \ , \qquad C_A^{(n)} = -\frac{g_A}{2} \ , \qquad g_A \sim 1.27$$

Neutrinos in hot/dense medium

ensemble of neutrinos described by incoherent mixture of states

neutrinos $\langle a^\dagger_{j,h'}(\vec{k}')\, a_{i,h}(\vec{k})\rangle \propto \delta^{(3)}(\vec{k}-\vec{k}')f^{ij}_{hh'}(\vec{k})$

antineutrinos $\langle b^\dagger_{j,h'}(\vec{k}')\, b_{i,h}(\vec{k})\rangle \propto \delta^{(3)}(\vec{k}-\vec{k}')\bar{f}^{ij}_{hh'}(\vec{k})$

$2n_f \times 2n_f$ matrix structure: Dirac case, need F and $\bar{F}$

$$F = \begin{pmatrix} f_{LL} & f_{L,R} \\ f_{R,L} & f_{RR} \end{pmatrix}$$

active-sterile coherence

$$\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{R,L} \\ \bar{f}_{L,R} & \bar{f}_{LL} \end{pmatrix}$$

Majorana case: $F = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix}$

neutrino-antineutrino coherence

Closed Time Path (CTP) formalism

2-point function:

$$G(x,y) = \left\langle T_{\mathrm{CTP}}\left(\Psi(x)\bar{\Psi}(y)\right)\right\rangle =: F(x,y) - \frac{i}{2}\rho(x,y)\,\mathrm{sgn}_{\mathrm{CTP}}(x^0 - y^0)$$

time-ordering along closed time path

ensemble average

statistical fct / occupation #: $F(x,y) = \frac{1}{2}\left\langle\left[\Psi(x), \bar{\Psi}(y)\right]\right\rangle$

spectral function: $\rho(x,y) = i\left\langle\left\{\Psi(x), \bar{\Psi}(y)\right\}\right\rangle$

Wigner transform: $F(X,k) = \int d^4r e^{ikr} F(X + \frac{1}{2}r, X - \frac{1}{2}r)$

Introduction to CTP: see e.g. Calzetta, Hu, *PRD* **37** (1988) 2878

Power counting / approximations

Assume neutrino masses, mass-splitting, matter potentials (induced by forward scattering), and external gradients are much smaller than neutrino energy:

$$m_\nu / E \sim \Delta m_\nu / E \sim \Sigma_{\text{forward}} / E \sim \partial_X / E \sim O(\epsilon)$$

$$\Sigma_{\text{inelastic}} / E \sim O(\epsilon^2)$$

i.e. assume physical quantities vary slowly on the scale of the neutrino de Broglie wavelength

QKEs include second order effects $\mathrm{O}(\epsilon^2)$

details: Vlasenko, Fuller, Cirigliano, *PRD* **89** (2014) 105004

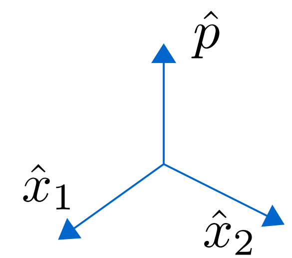
Projections

For ultra-relativistic neutrinos, it is useful to express all Lorentz tensors in terms of a basis formed by two light-like four-vectors and two transverse four-vectors

$$\hat{\kappa}^{\mu}(p) = (\mathrm{sgn}(p^0), \hat{p})\,, \qquad \hat{\kappa}'^{\mu}(p) = (\mathrm{sgn}(p^0), -\hat{p})\,, \qquad \hat{x}_{1,2}(p)\,,$$

$$\hat{x}^{\pm} \equiv \hat{x}_1 \pm i\hat{x}_2\,, \qquad \hat{\kappa}\cdot\hat{\kappa}' = 2 = -\hat{x}^{+}\cdot\hat{x}^{-}$$

The four independent spinor components of the Wigner Transform of the neutrino statistical two-point function are:



$$F_{L,R} = \frac{1}{4}\mathrm{Tr}\Big(\gamma_\mu P_{L,R}\, F(p,x)\Big)\,\hat{\kappa}^{\mu}$$

$$\Phi^{(\dagger)} = \mp\frac{i}{16}\mathrm{Tr}\Big(\sigma_{\mu\nu} P_{L/R}\, F(p,x)\Big)(\hat{\kappa}\wedge\hat{x}^{\pm})^{\mu\nu} e^{\pm i\varphi}$$

These can be collected in a $2n_f \times 2n_f$ matrix.

Quantum Kinetic Equations (QKE)

$$iDF = [H, F] + i\,\mathcal{C}$$

generalized "Vlasov" term

coherent evolution, generalizes MSW

collision term

$$H = \begin{pmatrix} H_R & H_m \\ H_m^\dagger & H_L \end{pmatrix},$$

H_m depends ***linearly*** on the neutrino mass

Spin flip sensitive to absolute mass scale!

details: Vlasenko, Fuller, Cirigliano, *PRD* **89** (2014) 105004;
Cirigliano, Fuller, Vlasenko, *Phys.Lett.* **B747** (2015) 27;
Vlasenko, Fuller, Cirigliano, arXiv:1406.6724

Collision term

$$\mathcal{C} = \frac{1}{2}\{\Pi^+, F\} - \frac{1}{2}\{\Pi^-, I - F\}$$

$$\Pi^\pm = \begin{pmatrix} \Pi_R^{\kappa\pm} & 2P^\pm \\ 2P^{\pm\dagger} & \Pi_L^{\kappa\pm} \end{pmatrix}$$

($2n_f \times 2n_f$ matrix)

$$F = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix}$$

(occupation # in diagonal, coherence in off-diagonal)

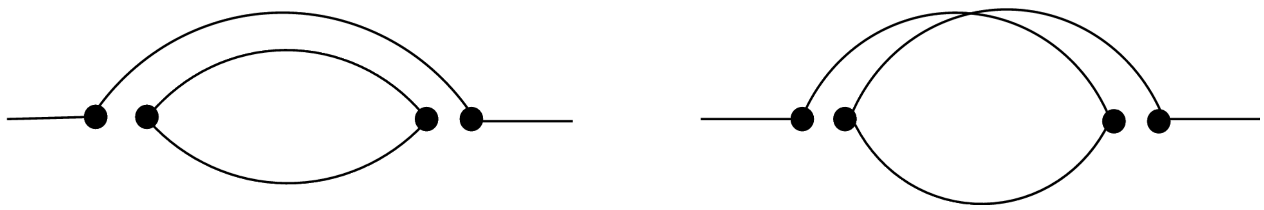
The collision term has a non-diagonal matrix structure in both flavor and spin space.

Results

DNB, V. Cirigliano, *in preparation*

Contributions to the collision term

Π :



- Neutrino-nucleon scattering processes
- Neutrino absorption and emission (charged-current processes)
- Neutrino-electron processes
- Neutrino-neutrino processes

only left topology

Example: NN-scattering

$$\Pi^{\pm}_{ab}(k) = -2G_F^2 \int \frac{d^4q_1\, d^4q_2\, d^4q_3}{(2\pi)^8} \delta^{(4)}(k - q_3 - q_1 + q_2)$$

$$\times \sum_{N=n,p} \Big\{ \gamma_\mu (P_L - P_R) G^{(\nu)\pm}_{ab}(q_3) \gamma_\nu (P_L - P_R)$$

$$\times \mathrm{Tr}\Big[\Gamma^\nu_N\, G^{(N)\mp}(q_2)\, \Gamma^\mu_N\, G^{(N)\pm}(q_1) \Big] \Big\}$$

$$G^{(N)+}(p) = 2\pi\delta(p^2 - m_N^2)(\not p + m_N)\Big[\theta(p^0)(1 - f(\vec{p})) - \theta(-p^0)\bar{f}(-\vec{p})\Big]$$

Neglect neutrino mass in these expressions because the collision term is already second order $\mathrm{O}(\epsilon^2)$

12 integrals and 7 delta functions

General expressions: amplitudes

Lorentz projections:

$$\Pi^{\pm}(k) = \begin{pmatrix} \Pi_R^{\kappa\pm}(k) & 2P^{\pm}(k) \\ 2P^{\pm\dagger}(k) & \Pi_L^{\kappa\pm}(k) \end{pmatrix}$$

$$= \frac{1}{2}\int d^4q_3 \begin{pmatrix} \underline{|A_-(q_3,k)|}^2(\bar{G}_V^L)^{\pm}(q_3) & A_-^{\dagger}(q_3,k)A_+(q_3,k)\Phi^{\pm}(q_3) \\ \underline{A_+^{\dagger}(q_3,k)A_-(q_3,k)}\Phi^{\pm\dagger}(q_3) & |A_+(q_3,k)|^2(\bar{G}_V^R)^{\pm}(q_3) \end{pmatrix}$$

- Can be written in terms of (square modulus of) amplitudes
- Generalizes earlier studies

Limit of k~q_3: compares to old results of L. Stodolski *PRD* **36** (1987) 2273

General expressions

$$\Pi = \frac{1}{2}\begin{pmatrix} |A_-|^2 \bar{G}_V^L & A_-^\dagger A_+ \Phi \\ A_+^\dagger A_- \Phi^\dagger & |A_+|^2 \bar{G}_V^R \end{pmatrix}$$

Majorana neutrinos:

$$P_{L/R}\,\psi(x) = \int \frac{d^3p}{2E(2\pi)^3}\left(u(p,\mp)a(p,\mp)e^{-ipx} + v(p,\pm)a^\dagger(p,\pm)e^{ipx}\right)$$

$$A = A_+ + A_-\,, \qquad A_\pm(q,p) = \pm\bar{u}(q,\pm)\gamma^\mu u(p,\pm)N_\mu$$

$$u(p,\pm)\bar{u}(p,\pm) = \not{p}P_{L/R}\,, \qquad u(p,\pm)\bar{u}(p,\mp) = \pm\frac{i}{4}Ee^{\pm i\varphi}(\hat{\kappa}\wedge\hat{x}^\pm)_{\mu\nu}\sigma^{\mu\nu}$$

Neutrino-neutrino interactions

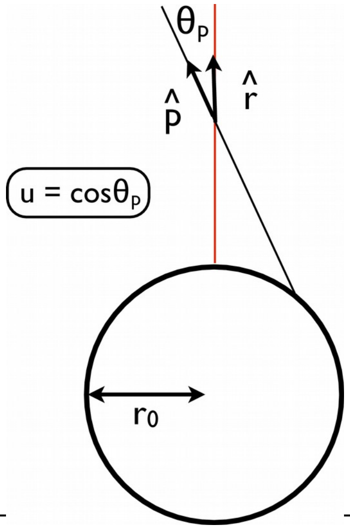
- “Wedges” appear also in the diagonal because of the neutrino “target”
- Always appear together with off-diagonal statistical fcts ϕ
- Up to four (instead of two) wedges can appear in the off-diagonal (drop out upon integrating the azimuthal angles in the special geometric cases we consider later)
- Will be interesting to plug collision-terms into QKEs numerically, but will need simplifying assumptions to be feasible

Approximations for supernovae

- Assuming spherical symmetry (in position space; “bulb model”), isotropic emittance of neutrinos, and time independence:
  all statistical functions (being Wigner transforms) depend only on

$$|\vec{k}|\,, \quad \theta_k\,, \quad |\vec{x}|$$

- Therefore can explicitly integrate over all φ_k
- Initially, have a total number of 9 integrals and 4 delta functions; integrating the azimuthal angles (in k-space) leaves us with 6 integrals and 3 delta functions.



Example: NN-scattering

$$\mathcal{C} = \begin{pmatrix} C & C_\phi \\ C_\phi^\dagger & \bar{C}^T \end{pmatrix}$$

neglecting anti-nucleons:

$$C = -G_F^2 \int \frac{r_1^3 r_2^3 r_3^3 dr_{1-3} d(\mathrm{cs}_{1-3})}{4(2\pi)^4 E_1 E_2 E_3} \delta(E_k - E_3 - E_1 + E_2)\delta(r_k - r_3 - r_1 + r_2)$$

$$\times \delta(\mathrm{cs}_k - \mathrm{cs}_3 - \mathrm{cs}_1 + \mathrm{cs}_2)\Bigg[\Bigg(\Big\{(1 - f_{N,1}) f_{N,2}(1 - f_3), f\Big\} - f \leftrightarrow (1 - f)\Bigg)$$

$$\times \Bigg((C_V - C_A)^2 (\tfrac{E_1}{r_1} - \mathrm{cs}_3\mathrm{cs}_1)(\tfrac{E_2}{r_2} - \mathrm{cs}_k\mathrm{cs}_2) - \frac{m_N^2}{r_1 r_2}(C_V^2 - C_A^2)(1 - \mathrm{cs}_k\mathrm{cs}_3)$$

$$+ (C_V + C_A)^2 (\tfrac{E_2}{r_2} - \mathrm{cs}_3\mathrm{cs}_2)(\tfrac{E_1}{r_1} - \mathrm{cs}_k\mathrm{cs}_1)\Bigg) \frac{E_3}{r_3}$$

$$+ 8(C_V^2 + C_A^2)\cos^2(\tfrac{\theta_k}{2}) \cos^2(\tfrac{\theta_3}{2}) \sin^2(\tfrac{\theta_1}{2}) \sin^2(\tfrac{\theta_2}{2})\Big((f_{N,2} - f_{N,1})\phi_3 \phi^\dagger + \mathrm{h.c.}\Big)\Bigg]$$

3-dim. Integrals left

Approximations for the early universe

- Assume all statistical functions depend only on the absolute values of the momenta (not their angles), spin coherence disappears.
- Therefore can explicitly integrate over all angles (e.g. following techniques of Dolgov, Hansen & Semikoz 1997).
- Initially, have a total number of 9 integrals and 4 delta functions; integrating all angles (in k-space) leaves us with 3 integrals and 1 delta function.
- Represents multi-flavor generalization of previous work.

→ ***Future: solve these 2-dim. Integrals numerically?***

Example: neutrino-neutrino processes

spin coherence disappears in early universe, therefore:

$$C = -\frac{G_F^2}{E_k^2}\int \frac{dE_1 dE_2 dE_3}{2\pi^3}\Bigg(\Big(E_1 E_3 D_2(E_1,E_3;E_2,E_k) + D_3(E_1,E_2,E_3,E_k)$$

$$+ E_2 E_k D_2(E_2,E_k;E_1,E_3) + E_1 E_2 E_3 E_k D_1(E_1,E_2,E_3,E_k)\Big)\times$$

$$\times\ \Big(\delta(E_k - E_3 - E_1 + E_2)\Big\{\Big(\mathrm{tr}((1-f_1)f_2) + (1-f_1)f_2\Big)(1-f_3), f\Big\}$$

$$+ \delta(E_k - E_3 + E_1 - E_2)\Big\{\Big(\mathrm{tr}\big(\bar f_1(1-\bar f_2)\big) + \bar f_1(1-\bar f_2)\Big)(1-f_3), f\Big\}$$

$$+ \delta(E_k + E_3 - E_1 - E_2)\Big\{\Big(\mathrm{tr}\big((1-f_1)(1-\bar f_2)\big) + (1-f_1)(1-\bar f_2)\Big)\bar f_3, f\Big\}\Big)$$

$$- f \leftrightarrow (1-f)$$

where D_i are polynomials in E_i (Dolgov, Hansen, Semikoz 1997)

→ *multi-flavor generalization*

Conclusion and Outlook

- ✔ Introduced and motivated the concept of QKEs
- ✔ Presented results for collision terms in the Majorana case
- ✔ Remaining integrals should be solvable numerically
- ✔ *To do:* generalize to Dirac neutrinos

References

1. D. N. Blaschke, V. Cirigliano, et al., *in preparation*

2. V. Cirigliano, G. Fuller, A. Vlasenko, *Phys.Lett.* **B747** (2015) 27

3. A. Vlasenko, G. Fuller, V. Cirigliano, *PRD* **89** (2014) 105004

4. A. Vlasenko, G. Fuller, V. Cirigliano, arXiv:1406.6724

Thank you for your attention!