

Neutrino and other fermion masses in domain-wall brane models

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Outline

- 1 Introduction
- 2 Review
- 3 The SU(5) model
- 4 See-saw mechanism for DW neutrinos
- 5 Summary

References

- Work performed with Rhys Davies, Damien George and Ben Callen.
- DGV, Phys. Rev. D77, 124038 (2008); arXiv:0705.1584 [hep-ph].
- CV, work-in-progress.

Context

Varieties of brane-world models:

- String-theoretic.
- Field-theoretic with brane(s) as δ -fn object(s) put in by hand. Often taken as effective low-energy outcomes of string theory, e.g. RS1.
- Completely field-theoretic, with brane as soliton (domain wall, vortex, domain-wall junctions).

I shall discuss this last class. Specifically, we'll have one topologically-infinite extra dimension (like RS2) and the brane will be a domain wall.

Motivations

Why be interested in this class of brane-worlds?

- We know that field-theory describes particle physics very well.
- It is a bottom-up approach, contrasting with the top-down string-theory philosophy.
- Is there a phenomenologically-viable model in this class?
- Once you have a candidate for a viable model, does it solve any problems?

Outcomes

I will present evidence that viable models may exist, and that they can help solve the fermion mass hierarchy puzzle without using a horizontal symmetry.

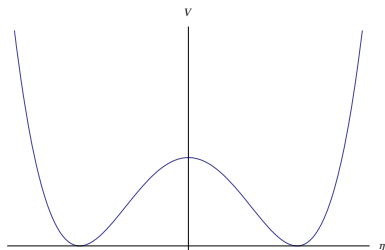
We should keep an open mind about whether this important problem has a symmetry resolution or some other explanation.

Turns out one is driven to GUT realisations. Two of the usual problems of GUTs,

- unwanted mass relations such as $m_e = m_d$,
- coloured-Higgs induced proton decay,

can be avoided without complicating the theory with epicycles.

Domain walls and kinks



We need a potential with *disconnected* and *degenerate* vacua:

$$V = \lambda(\eta^2 - v^2)^2$$

with η a scalar field.

Domain walls and kinks

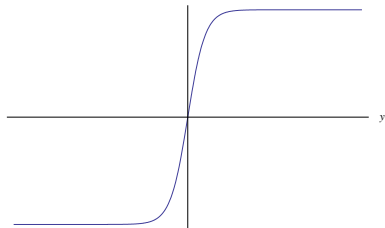
Lagrangian for $\eta(x^\mu, y)$:

$$\mathcal{L} = -\frac{1}{2}\partial_M\eta\partial^M\eta - V(\phi)$$

A solution is the *kink*:

$$\eta_{\text{kink}}(y) = v \tanh(\sqrt{2\lambda}vy)$$

It is topologically stable.



Fermion localisation

Let $\Psi(x, y)$ be 5d fermion Yukawa-coupled to background scalar field $b(y)$. It obeys 5d Dirac Eq:

$$i\Gamma^M \partial_M \Psi - b(y)\Psi = 0$$

where $\Gamma^M = (\gamma^\mu, -i\gamma_5)$.

Do mode decomposition (generalised KK expansion):

$$\Psi(x, y) = \sum_m [f_L^m(y)\psi_L^m(x) + f_R^m(y)\psi_R^m(x)]$$

The ψ 's are 4d fermions, forced to obey 4d Dirac Eq:

$$i\gamma^\mu \partial_\mu \psi_{L,R}^m = m\psi_{R,L}^m$$

Fermion localisation

The mode functions $f_{L,R}^m$ then obey the Schrödinger-like equations

$$-f_{L,R}^{m\prime\prime} + W_{\mp} f_{L,R}^m = m^2 f_{L,R}^m$$

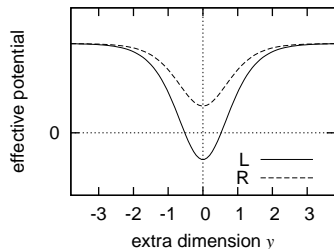
with effective potentials

$$W_{\mp}(y) = b(y)^2 \mp b'(y).$$

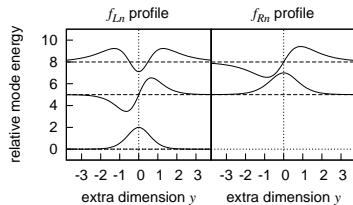
At points y_0 s.t. $b(y_0) = 0$, you get a *localised chiral zero mode*. The chirality depends on whether $b(y)$ slopes up (LH) or down (RH).

These are our candidate quarks and leptons.

Fermion localisation



Fermion trapping potential



Fermion modes

Fermion localisation

The profile for the chiral zero mode is:

$$f(y) \propto e^{-\int_{y_0}^y b(y') dy'}$$

The b 's depend on the Yukawas, so the f 's are *exponentially sensitive* to them. This is the key to getting fermion mass hierarchies (see later).

Mode functions are normalised as per $\int_{-\infty}^{+\infty} f(y)^2 dy = 1$ to make 4d field kinetic terms conventionally normalised.

Spin-0 boson localisation

This proceeds in a very similar way to fermions. I'll omit the details here.

One important outcome: a localised 4d scalar can have a tachyonic mass-squared. Hence we can have SSB inside the wall. We'll use this for the electroweak Higgs doublet.

Graviton localisation

A trivial modification of type-2 Randall-Sundrum.

There are some tricky things with how this affects the localisation of other kinds of particles.

All is OK, but it requires a long discussion.

For simplicity, I'll stick to flat space here.

Gauge field localisation

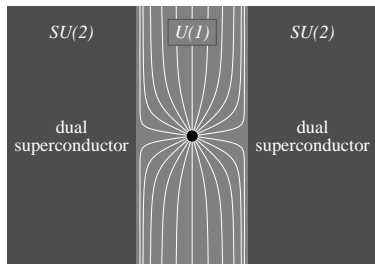
The Dvali-Shifman mechanism requires:

- 1 The unbroken symmetry inside the wall to be a subgroup of the bulk symmetry.
- 2 The bulk to be in confinement phase.

Unlike other types of field localisation, this mechanism relies on non-perturbative QFT rather than classical FT.

It is not firmly established, but it is plausible. Assuming it works allows model-building to progress!

Gauge-field localisation



The confining bulk repels the field lines of a source inside the wall. The large distance gauge field behaviour inside the wall is effectively dimensionally-reduced by 1.

The background DW

Dvali-Shifman tells us that we should embed $SU(3) \times SU(2) \times U(1)$ in a larger group that breaks to it inside the wall.

The minimal sensible choice is $SU(5)$.

We use an $SU(5)$ singlet scalar η to produce a kink, and an $SU(5)$ adjoint χ to break $SU(5)$ to the SM inside the wall.

Write $\chi = \sum_a T^a \chi_a$, where T 's are $SU(5)$ generators in the fundamental. If the component χ_1 corresponding to the hypercharge generator Y condenses, then $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)_Y$.

The background DW

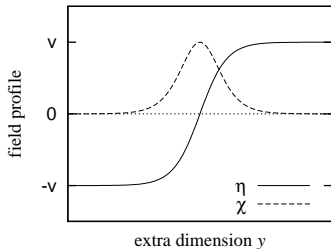
You write a Higgs potential, arrange the global minima

$$\langle \eta \rangle = \pm v, \quad \langle \chi \rangle = 0,$$

use them as boundary conditions, solve the Euler-Lagrange equations to get, e.g.

$$\eta(y) = v \tanh(ky),$$

$$\chi_1(y) = A \operatorname{sech}(ky).$$



This simple analytical solution holds on a certain parameter slice. Off that slice, similar solutions exist but must be obtained numerically.

Fermion localisation

Next, you introduce 5d fermions

$$\Psi_5 \sim 5^*, \quad \Psi_{10} \sim 10, \quad N \sim 1$$

and you Yukawa-couple them to η and χ :

$$\begin{aligned} Y_{DW} &= h_{5\eta} \bar{\Psi}_5 \Psi_5 \eta + h_{5\chi} \bar{\Psi}_5 \chi^T \Psi_5 \\ &+ h_{10\eta} \text{Tr}(\bar{\Psi}_{10} \Psi_{10}) \eta - 2h_{10\chi} \text{Tr}(\bar{\Psi}_{10} \chi \Psi_{10}) \\ &+ h_{1\eta} \bar{N} N \eta. \end{aligned}$$

The background fields you use in the 5d Dirac Eq. are:

$$b_{nY}(y) \equiv h_{m\eta} \eta(y) + \sqrt{\frac{3}{5}} \frac{Y}{2} h_{n\chi} \chi_1(y).$$

SM components of different hypercharge Y couple to different linear combinations of $\eta(y)$ and $\chi_1(y)$. Fermions are *split*, but not arbitrarily.

EW symmetry breaking

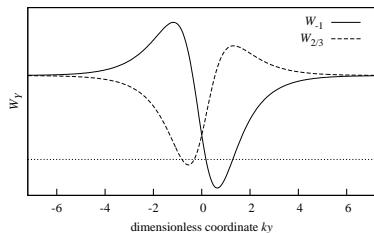
Now introduce a scalar $\Phi \sim 5^*$ containing the weak doublet Φ_w and a coloured scalar Φ_c . Yukawa-couple it to fermions in the usual way.

You do a mode decomposition, and are interested in the lowest modes:

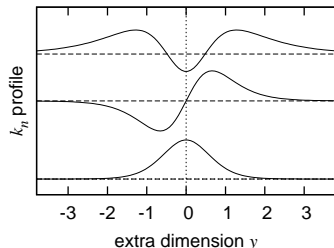
$$\Phi_{w,c}(x, y) = p_{w,c}(y)\phi_{w,c}(x)$$

You write the Higgs potential, plug the above into the Euler-Lagrange Eqs., get effective Schrödinger Eqs. for the profiles $p(y)$.

EW symmetry breaking



Scalar trapping potentials



Scalar modes

The p_w well is deeper (due to parameter region chosen) and gets a negative value m_w^2 triggering spontaneous EW symmetry breaking.

Fermion spectra

I now report some preliminary results from work by Ben Callen on fitting the model to the observed quark and lepton masses, including neutrinos.

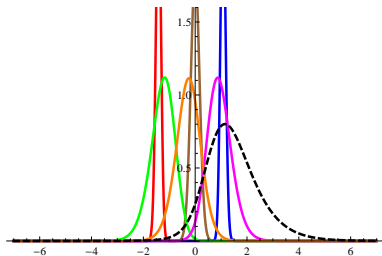
A 4d Yukawa coupling is of the form:

$$h \left[\int dy f_L(y) f_R(y) p(y) \right] \bar{\psi}_L(x) \psi_R(x) \phi(x).$$

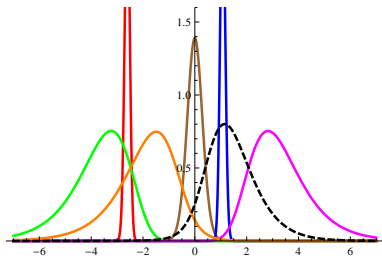
The 4d Yukawa coupling constant is equal to the 5d Yukawa multiplied by an overlap integral of profile functions, which themselves depend on Yukawas in a complicated way.

The profiles are *exponentially sensitive* to the Yukawa coupling constants. Searching the parameter space is numerically intensive. We have been proceeding by trial-and-error. Multiple viable regions exist.

Fermion spectra – no mixing



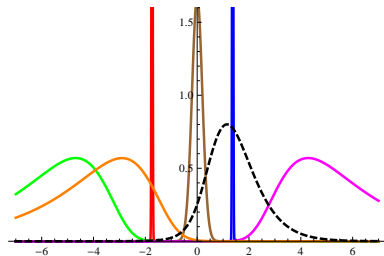
3rd gen profiles



2nd gen profiles

Brown - the right handed neutrinos; Red - the left handed lepton doublets; Blue - right handed down, strange, and bottom quarks; Green - right handed electron, muon and tau; Magenta - right handed up, charm, and top quarks; Orange - the left handed quark doublets; Black dashed - the Electroweak Higgs.

Fermion spectra – no mixing



1st gen profiles

All EW Yukawas have been set as equal!

Fermion mass differences *entirely* driven by profiles through coupling to the DW background. The masses can be fitted well, including very light Dirac neutrinos.

About a 5 order-of-mag spread used in fermion-DW Yukawas. This will be reduced by making EW Yukawas unequal. Fermion mass spread is 14 orders-of-mag.

Higgs-induced proton decay suppression

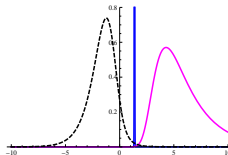
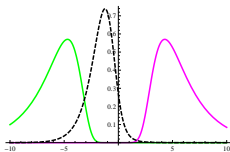
This process proceeds via the Yukawa terms

$$\bar{u}_R(e_R)^c \phi_C^* \quad \text{and} \quad \bar{d}_R(u_R)^c \phi_C.$$

For the *same* region of parameter space that fits the masses, the effective 4d Yukawa couplings constants are, respectively, about

$$10^{-20} \quad \text{and} \quad 10^{-5}.$$

The proton partial lifetime goes below the experimental bound for ϕ_C mass greater than several TeV.



Summary

- The fermion mass spectrum can definitely be fitted, including for Dirac neutrinos, despite the minimal $SU(5)$ structure.
- A 5 order-of-mag Yukawa spread gives a 14 order-of-mag fermion mass spread.
- CKM mixing angles can almost certainly be fitted as well.
- Higgs induced proton decay automatically suppressed. Elimination of doublet-triplet splitting problem.

Open questions:

- Optimisation of the fit, and discovery of all allowed regions of parameter space.
- As part of that, reducing the hierarchy of fermion-DW couplings as much as possible. It looks like a 2-3 order-of-mag range should be possible.
- How *generic* is a strong mass hierarchy in this picture?

DW Majorana fermions

The need to get extremely light Dirac neutrinos was a major driver in the need to have DW-fermion couplings spanning a 5 order-of-mag range.

This requires more study, our results are *preliminary*, but it does motivate seeing if the see-saw mechanism can work for DW neutrinos.

Now, if you just add a bare 5d Dirac mass to the Lagrangian, it does *not* give the zero mode a mass: it just shifts its localisation position. This is the split fermion idea of Arkani-Hamed and Schmaltz.

What happens if you add a 5d Majorana mass? ($N^c = \Gamma^5 \Gamma^2 N^*$.)

$$\mathcal{L} = i\bar{N}\Gamma^M\partial_M N + h_{1\eta}\bar{N}N\eta - \frac{1}{2}m\left(\bar{N}N^c + \bar{N}^c N\right)$$

DW Majorana fermions

We found that in this case there is no zero-mode.

The would-be zero mode becomes a 4d Majorana fermion of mass m .

The profile function is the same as for the zero-mode, multiplied by an uninteresting phase.

Hence, we conclude that the see-saw mechanism can be implemented for DW neutrinos.

Summary

- Viable domain-wall brane models may exist. The main issue is the Dvali-Shifman mechanism.
- They can alleviate the fermion hierarchy problem. A 5 order-of-mag spread in fermion-DW Yukawas gives you a 14 order-of-mag spread in fermion masses.
- You can get extremely light Dirac neutrinos.
- You can get localised Majorana fermions, so the see-saw mechanism can be implemented for DW neutrinos.
- They are GUT models without the usual problems of incorrect mass relations and doublet-triplet splitting.